Classical Solutions and Order of Zeros in Open String Field Theory

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Abstract

Earlier an analytic approach is proposed for classical solutions describing tachyon vacuum in open string field theory. Based on the approach, we construct a certain class of classical solutions written in terms of holomorphic functions with higher order zeros. Taking the simplest among the new classical solutions, we study the cohomology of the new BRS charge and make a numerical analysis of the vacuum energy. The results indicate that the new non-trivial solution is another analytic candidate for the tachyon vacuum.

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1 Introduction

Over the past years, it has been argued that string field theory, originally defined around the D-brane vacuum, admits the lower energy vacuum where the D-brane is annihilated via the condensation of the open string tachyon [1, 2]. This should be described by a classical solution for which the value of the action is the minus of the tension of the annihilated brane [3, 4]. The classical solution in cubic open string field theory (CSFT) [5] has been constructed numerically using the level truncation method in the Siegel gauge [6, 7, 8, 9]. These studies provide us with sufficiently accurate quantitative tests of the above conjecture.

Remarkably, there exists a candidate for analytic description of the tachyon condensed vacuum as classical solutions in CSFT. In [10], one parameter family of classical solutions in pure gauge form has been constructed in a half string formulation by taking certain linear combinations of the BRS current and the ghost field acting on the identity operator. A linear combination is specified by a holomorphic function $h_a(w)$ with a real parameter a. The solutions are expressed on the universal basis: the matter Virasoro operators, the ghost and anti-ghost fields. The work [10] is extended in [11] to give more general solutions by considering infinite set of one parameter families of holomorphic functions $h_a^m(w)$ labelled by a positive integer m. The solutions are in the pure gauge form and mostly reduced to gauge transformations of the usual vacuum. However non-trivial solutions emerge when the parameter in $h_a^m(w)$ takes the boundary value $a = a_b$. Thus, we have a series of non-trivial classical solutions labelled by the integer m.

Given a solution with $h_a^m(w)$, we may expand the action around the solution to have a theory for the string fluctuating around the classical solution. In CSFT, the new action remains the same form, but the BRS charge Q_B is replaced by the new one (Q_B'), which depends on the function h_a^m. Study on the cohomology defined by Q_B' [11], perturbative scattering amplitudes [12] and a numerical analysis of the vacuum energy [13] strongly indicate that the non-trivial solutions correspond to the tachyon vacuum.

It is found that properties of the analytic solutions depend on the structure of zeros in the underlying function $F(w) = e^{h(w)}$ [14, 15]: the functions $F_a^m(w)$ for $a \neq a_b$ have $4m$ first-order zeros inside as well as outside of the unit circle $|w| = 1$. In the limit $a \to a_b$, each pair of zeros, one inside and the other outside, becomes degenerate on the unit circle. The functions $F_{a_b}^m(w)$ turn out to have $2m$ second-order zeros on the unit circle, and produce non-trivial solutions which cannot be gauged away. The implications of the structure of zeros in the function $F(w)$ remain to be clarified. One asks
(1) how the solutions with $F_{a_b}^l$ and $F_{a_b}^m$ ($l \neq m$), which differ in locations and numbers of zeros, could be related,
(2) if some new aspects appear in another class of solutions associated with functions having

higher order zeros, in particular, if those solutions give rise to different values for the vacuum energy, reflecting the order of zeros.

The purpose of this paper is to consider the above question (2). We construct a classical non-trivial solution to CSFT characterized by a holomorphic function with fourth order zeros, and discuss its properties in details. After obtaining explicit form of the solution, we examine cohomology of the new BRS charge, the kinetic operator for shifted string field around the solution. Numerical analysis of the vacuum energy is also reported. Our results indicate that the new non-trivial solution with higher order zeros provides us with yet another analytic candidate for the tachyon vacuum.

2 Classical Solutions in Open String Field Theory

Let us briefly summarize the approach in Ref. [10] and its results on classical solutions in CSFT. The action of CSFT [5] is given by

$$S[\Psi] = -\frac{1}{g^2} \int \left(\frac{1}{2} \Psi * Q_{\mathrm{B}} \Psi + \frac{1}{3} \Psi * \Psi * \Psi \right), \tag{2.1}$$

where Ψ is the string field and Q_{B} is the Kato-Ogawa BRS charge [16]. This action leads to the equation of motion,

$$Q_{\mathrm{B}} \Psi + \Psi * \Psi = 0. \tag{2.2}$$

The classical solutions are written in terms of two basic operators, the BRS current $J_{\mathrm{B}}(w)$ and the ghost field $c(w)$, acting on the identity operator I. A solution is given by a contour integral of a linear combination of these operators specified by a holomorphic function $h(w) = \log F(w)$ [10]:

$$\begin{aligned}
\Psi_0(h) &= \int_{C_{\mathrm{left}}} \frac{dw}{2\pi i} \left[\left(e^h - 1 \right) J_{\mathrm{B}}(w) - \left((\partial h)^2 e^h \right) c(w) \right] I \\
&\equiv Q_{\mathrm{L}}(e^h - 1)I - C_{\mathrm{L}} \left((\partial h)^2 e^h \right) I.
\end{aligned} \tag{2.3}$$

Here, the integral is taken along the left-half of a circle, as indicated by C_{left} [10]. The function $h(w)$ is required to satisfy two conditions, (i) $h(-1/w) = h(w)$ and (ii) $h(\pm i) = 0$. The first condition is necessary to ensure the basic symmetry property of CSFT under the inversion and the second one guarantees the closure of the half-splitting algebra.

Expanding the original string field around the classical solution,

$$\Psi = \Psi_0(h) + \Phi, \tag{2.4}$$

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we rewrite the action as

$$S[\Psi] = S[\Psi_0(h)] - \frac{1}{g^2} \int \left(\frac{1}{2} \Phi * Q'_{\mathrm{B}} \Phi + \frac{1}{3} \Phi * \Phi * \Phi \right). \tag{2.5}$$

Note here that the three-string vertex for the string field Φ remains the same form as the original one, while the kinetic term carries the new BRS charge given by

$$Q'_{\mathrm{B}} \Phi \equiv Q_{\mathrm{B}} \Phi + \Psi_0(h) * \Phi + \Phi * \Psi_0(h). \tag{2.6}$$

More explicitly, it is written as

$$\begin{aligned} Q'_{\mathrm{B}} &= \oint \frac{dw}{2\pi i} \left[e^h J_{\mathrm{B}}(w) - (\partial h)^2 e^h c(w) \right] \\ &\equiv Q(e^h) - C\left((\partial h)^2 e^h \right), \end{aligned} \tag{2.7}$$

where the integration is taken over a closed circle or $C_{\mathrm{left}} + C_{\mathrm{right}}$: this is due to the contributions from the second and third terms of eq. (2.6), in which the classical solution is multiplied on the left and right halves of the string field Ψ.

In [11], the universal solutions constructed with a set of functions

$$h_a^m(w) = \log \left\{ 1 - \frac{a}{2} (-1)^m \left(w^m - \left(-\frac{1}{w} \right)^m \right)^2 \right\} \quad (m = 1, 2, 3, \cdots) \tag{2.8}$$

were discussed. The parameter a must be $a \geq -1/2$ to satisfy the hermiticity requirement. Exponentiating these functions, we have

$$\begin{aligned} F_a^m(w) &= \exp\left(h_a^m(w) \right) = \frac{\left(1 - (-1)^m Z(a) \, w^{2m} \right)\left(1 - (-1)^m Z(a) \, w^{-2m} \right)}{(1 - Z(a))^2}, \\ Z(a) &\equiv \frac{1 + a - \sqrt{1 + 2a}}{a}. \end{aligned} \tag{2.9}$$

For $a > -1/2$, $Z(a) \in (-1, \ 1)$ and $Z(-1/2) = -1$.

For each function $h_a^m(w)$, we have shown the followings for $a > -1/2$:

1. The action obtained by expanding around the solution can be transformed back to the action with the original BRS charge [10];

2. The new BRS charge gives rise to the cohomology which has one-to-one correspondence to the cohomology of the original BRS charge [11];

3. The expanded theory reproduces the same open string scattering amplitudes as the original theory [12];

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4. We can also show numerically that the expanded theory has a non-perturbative vacuum and its vacuum energy tends to the value appropriate to cancel the D-brane tension as the truncation level increases [13].

All these facts are consistent with the expectation that solutions for $a > -1/2$ are trivial pure gauge solutions. On the other hand, around a solution with $a = -1/2$, we find completely different properties in the expanded theory:

5. The new BRS charge has the vanishing cohomology in the Hilbert space of the ghost number one [11];

6. Any open string scattering amplitudes vanish and perturbatively there are no open string excitations (*no open string theorem*) [12];

7. A numerical analysis shows that the non-perturbative vacuum found for $a > -1/2$ disappears as a approaches to $-1/2$ [13].

Hence, we believe that the non-trivial solutions correspond to the tachyon vacuum.

The above results are related to the distribution and the order of zeros in the functions $\exp(h_a^m(w)) = F_a^m(w)$. Consider the function $F_a^1(w)$, for example. For $a > -1/2$, it has the first-order zeros at $w = \pm\sqrt{-Z(a)}, \pm 1/\sqrt{-Z(a)}$. For $a = -1/2$, $Z(-1/2) = -1$ and the zeros at $\sqrt{-Z(a)}$ and $-\sqrt{-Z(a)}$ coincide with those at $1/\sqrt{-Z(a)}$ and $-1/\sqrt{-Z(a)}$, respectively: In other words, these points become the second-order zeros. We have the non-trivial solution when this change in the distribution of zeros occurs. In general, the function $F_a^m(w)$ has $4m$ first-order zeros for $a > -1/2$. Half of them are distributed inside the unit circle $|w| = 1$ while the other half are located outside of it, as a result of the condition $h_a^m(w) = h_a^m(-1/w)$. In the limit of $a \to -1/2$, these $4m$ first-order zeros merge into $2m$ second-order zeros on the unit circle and non-trivial solutions emerge.

From these observations, we wonder if the order of zeros has any relevance in characterizing non-trivial solutions. Actually, it has been conjectured that the order is related to the number of D-branes [14]. So it would be worthwhile to consider other solutions written in terms of holomorphic functions with higher oder zeros.

3   Classical Solutions with Higher Order Zeros

We now construct a new family of classical solutions. The underlying holomorphic function is expressed as

$$h_{\{a_m\}}(w) = \log\left[1 + \sum_m a_m\Big(\exp h_{-1/2}^m(w) - 1\Big)\right]. \qquad (3.1)$$

where $h_a^m(w)$ is given by eq. (2.8). Hence, as required for classical solutions, $h_{\{a_m\}}(w)$ satisfies $h_{\{a_m\}}(-1/w) = h_{\{a_m\}}(w)$ and $h_{\{a_m\}}(\pm i) = 0$. The classical solutions associated with $h_{\{a_m\}}(w)$ are parameterized by $\{a_1, a_2, \cdots\}$, which can be chosen in such a way that new solutions with higher order zeros appear.

For example, consider the function made of $h^1_{-1/2}$ and $h^2_{-1/2}$,

$$h_{\{a_1, a_2\}}(w) = \log \left[1 + a_1 \left\{ -\frac{1}{4} \left(w - \frac{1}{w} \right)^2 - 1 \right\} + a_2 \left\{ \frac{1}{4} \left(w^2 + \frac{1}{w^2} \right)^2 - 1 \right\} \right]. \tag{3.2}$$

Choosing $a_1 = 4a_2 = -2a$, we obtain one parameter family of functions,

$$h_a^{(4)}(w) \equiv h_{\{-2a, -a/2\}}(w) = \log \left\{ 1 + 2a - \frac{a}{8} \left(w - \frac{1}{w} \right)^4 \right\}. \tag{3.3}$$

The parameter a is larger than or equal to $-1/2$ due to the hermiticity condition of the classical solution. For $a = -1/2$, we find

$$\exp h_{-1/2}^{(4)}(w) = \frac{1}{16} \left(w - \frac{1}{w} \right)^4, \tag{3.4}$$

which has fourth order zeros at $w = \pm 1$.

The function $\exp h_a^{(4)}(w)$ can be rewritten as

$$\exp h_a^{(4)}(w) = \frac{1}{(1+x)^2(1+y)^2}(1 - xw^2)(1 - xw^{-2})(1 - yw^2)(1 - yw^{-2}), \tag{3.5}$$

where the parameters x and y are related to a by

$$a = -\frac{8xy}{(1+x)^2(1+y)^2}, \qquad (x+y)\left(1 + \frac{1}{xy}\right) = 4. \tag{3.6}$$

Since the function (3.5) is invariant under the transformations $x \to 1/x$ and $y \to 1/y$, we can impose the condition $|x| \leq 1$ and $|y| \leq 1$ on the parameters x and y. It follows from eq. (3.6) that $|x| < 1$ and $|y| < 1$ for $a > -1/2$, and $x = y = 1$ for $a = -1/2$. Using x and y, we can expand the function $h_a^{(4)}(w)$ in a Laurent series,

$$h_a^{(4)}(w) = -\log(1+x)^2(1+y)^2 - \sum_{n=1}^{\infty} \frac{1}{n}(x^n + y^n)(w^{2n} + w^{-2n}). \tag{3.7}$$

In the theory expanded around the solution constructed with $h_a^{(4)}(w)$, the new BRS charge is given by

$$Q_{\mathrm{B}}'(a) = Q(e^{h_a^{(4)}}) - C\left((\partial h_a^{(4)})^2 e^{h_a^{(4)}}\right). \tag{3.8}$$

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We now examine if this charge $Q_{\mathrm{B}}'(a)$ is related to the original Q_{B} via a similarity transformation. Defining the operator

$$q(f) = \oint \frac{dw}{2\pi i} \, f(w) \, J_{\mathrm{gh}}(w),$$ (3.9)

and using the commutation relations [10]

$$\begin{aligned}
[q(f), \, Q(g)] &= Q(fg) - 2C(\partial f \partial g), \\
[q(f), \, C(g)] &= C(fg),
\end{aligned}$$ (3.10)

formally we obtain

$$Q(e^{h_a^{(4)}}) - C\left((\partial h_a^{(4)})^2 e^{h_a^{(4)}}\right) = e^{q(h_a^{(4)})} \, Q_{\mathrm{B}} \, e^{-q(h_a^{(4)})}.$$ (3.11)

This equality holds of course only if the operator $\exp q(h_a^{(4)})$ is well-defined. Using (3.7) and the mode expansion $J_{\mathrm{gh}}(w) = \sum q_n w^{-n-1}$, we expand the operator $q(h_a^{(4)})$ as

$$q(h_a^{(4)}) = -\log(1+x)^2(1+y)^2 \, q_0 - \sum_{n=1}^{\infty} \frac{1}{n} \, (x^n + y^n)(q_{2n} + q_{-2n}).$$ (3.12)

We find that the normal ordered form of $\exp q(h_a^{(4)})$ has a singularity at $a = -1/2$, because the commutation relation between positive and negative modes of $q(h_a^{(4)})$ diverges at $a = -1/2$ as follows,

$$\left[q^{(+)}(h_a^{(4)}), \, q^{(-)}(h_a^{(4)})\right] = 2\sum_{n=1}^{\infty} \frac{1}{n}(x^n + y^n)^2 \to 8\sum_{n=1}^{\infty} \frac{1}{n} = \infty \quad (a \to -1/2).$$ (3.13)

Thus, (3.11) holds for $a > -1/2$, but not for $a = -1/2$: the expanded theory for $a > -1/2$ can be transformed back to the original theory by the string field redefinition $\Psi' = \exp q(h_a^{(4)}) \times \Psi$, but this is not the case for $a = -1/2$. The conclusion here is parallel to that for the case of the function (2.8) [10, 11].

It follows from the above argument that the function $h_{-1/2}^{(4)}(w)$ generates a new non-trivial solution with fourth order zeros. The solution has a well-defined universal Fock space expression as for the case of $h_a^m(w)$ [10, 11]. Actually, substituting the expressions

$$\begin{aligned}
e^{h_{-1/2}^{(4)}(w)} - 1 &= \frac{1}{16}(w^4 + w^{-4}) - \frac{1}{4}(w^2 + w^{-2}) - \frac{5}{8}, \\
\left(\partial h_{-1/2}^{(4)}(w)\right)^2 e^{h_{-1/2}^{(4)}(w)} &= \frac{(1+w^2)^2(1-w^2)^2}{w^6}
\end{aligned}$$ (3.14)

into (2.3) and performing the integrations, we obtain

$$\begin{aligned}
|\Psi_0(-1/2)\rangle &= \frac{1}{8\pi} \sum_{n=0}^{\infty} (-1)^n \left[\frac{1}{2n+5} + \frac{4}{2n+3} - \frac{10}{2n+1} + \frac{4}{2n-1} + \frac{1}{2n-3}\right] Q_{-2n-1} |I\rangle \\
&\quad + \frac{8}{3\pi} c_1 |I\rangle + \frac{8}{5\pi} c_{-1} |I\rangle \\
&\quad - \frac{2}{\pi} \sum_{n=1}^{\infty} (-1)^n \left[\frac{1}{2n+5} + \frac{1}{2n-3} - \frac{2}{2n+1}\right] c_{-2n-1} |I\rangle,
\end{aligned}$$ (3.15)

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where the mode expansions are given by $J_{\mathrm{B}}(w) = \sum Q_n w^{-n-1}$ and $c(w) = \sum c_n w^{-n+1}$.

In the following, we will study the classical solutions obtained with $h_a^{(4)}(w)$ in detail to provide physical interpretation of the solution with higher order zeros. We first discuss the cohomology of the new BRS charge (3.8), and then the vacuum energy of a non-perturbative vacuum.

3.1   Cohomology

For $a > -1/2$, the similarity transformation (3.8) relates the new BRS to the original BRS charge. Then, the cohomology of the new BRS charge is given by the similarity transformation of the original cohomology [16, 17]: the state $|\psi\rangle$ satisfying $Q'_{\mathrm{B}}(a)|\psi\rangle = 0$ ($a > -1/2$) can be written as

$$|\psi\rangle = e^{q(h_a^{(4)})} \left(|P\rangle \otimes c_1 |0\rangle + |P'\rangle \otimes c_0 c_1 |0\rangle\right) + Q'_{\mathrm{B}}(a)|\phi\rangle, \qquad (3.16)$$

where $|P\rangle$ and $|P'\rangle$ are positive norm states in the matter sector. Thus, there exists one-to-one correspondence between the cohomologies of the new and original BRS charges for $a > -1/2$.

For $a = -1/2$, the new BRS charge is given by

$$
\begin{aligned}
Q'_{\mathrm{B}}(-1/2) &= Q\left(\frac{1}{16}\left(w - \frac{1}{w}\right)^4\right) - C\left(w^{-2}\left(w^2 - \frac{1}{w^2}\right)^2\right) \\
&= \frac{3}{8}Q_{\mathrm{B}} - \frac{1}{4}(Q_2 + Q_{-2}) + \frac{1}{16}(Q_4 + Q_{-4}) + 2c_0 - c_4 - c_{-4}. \qquad (3.17)
\end{aligned}
$$

Using the first commutation relation in (3.10), we can transform the new BRS charge to a level four operator as

$$e^{q(f)} Q'_{\mathrm{B}} e^{-q(f)} = \frac{1}{16}(Q_4 - 16c_4), \qquad (3.18)$$

where $f(w)$ and $q(f)$ are given by

$$f(w) = -4\log(1 - w^{-2}), \quad q(f) = 4\sum_{n=1}^{\infty}\frac{1}{n}q_{-2n}. \qquad (3.19)$$

Using eq. (3.18), we may find the cohomology for the case of $a = -1/2$ in the same way as that for $h_a^m(w)$ [11]. First, the cohomology of the operator $Q_4 - 16c_4$ becomes

$$|P\rangle \otimes b_{-4}b_{-3}b_{-2}|0\rangle + |P'\rangle \otimes b_{-3}b_{-2}|0\rangle + (Q_4 - 16c_4)|\phi\rangle. \qquad (3.20)$$

This is due to the fact that $Q_4 - 16c_4$ may be obtained from the original BRS charge Q_{B}, by simply replacing the ghost oscillators c_n and b_n by c_{n+4} and b_{n-4} without changing their orders [11]. Therefore, the state satisfying $Q'_{\mathrm{B}}(-1/2)|\psi\rangle = 0$ can be written as

$$|\psi\rangle = |P\rangle \otimes e^{-q(f)}b_{-4}b_{-3}b_{-2}|0\rangle + |P'\rangle \otimes e^{-q(f)}b_{-3}b_{-2}|0\rangle + Q'_{\mathrm{B}}(-1/2)|\phi\rangle. \qquad (3.21)$$

The ghost numbers of the cohomologically non-trivial states are -3 and -2, which differ from that of the original BRS charge. Hence, we conclude that there are no open string excitations perturbatively in the theory expanded around the non-trivial solution with fourth order zeros.

3.2 Vacuum energy

The classical solution with $h^{(4)}_{-1/2}(w)$ is a non-trivial solution as indicated by the cohomological analysis of the new BRS charge. Accordingly, the vacuum energy of the solution is expected to have a non-zero value. A solution with $a > -1/2$ is a pure gauge solution and its vacuum energy must be zero.

This resembles to the situation encountered in Ref. [13]. So, it would be useful to recall what had happened in earlier case. There, due to some technical reasons, it was found difficult to directly evaluate the energy density of our solution. So rather than calculating it directly, the theory expanded around a solution was taken and the energy of a non-perturbative vacuum of that theory was evaluated. The action of the expanded theory is given by

$$S[\Phi] = -\frac{1}{g^2} \int \left(\frac{1}{2} \Phi * Q'_{\mathrm{B}}(a)\Phi + \frac{1}{3}\Phi * \Phi * \Phi \right), \tag{3.22}$$

where the new BRS charge is given as eq.(3.8). The action (3.22) with a pure gauge solution is the same as the original theory and it observes the tachyon vacuum as its non-perturbative vacuum. Therefore the vacuum energy takes a non-zero value. On the other hand, when expanded around the non-trivial vacuum, the theory described by the action (3.22) is already at the tachyon vacuum and the vacuum energy is zero. In summary, by denoting the vacuum solution as $\Phi_0(a)$, the vacuum energy density behaved as

$$f_a(\Phi_0(a)) \equiv -S[\Phi_0(a)]/T_D V_D = \begin{cases} -1 & (a > -1/2) \\ 0 & (a = -1/2), \end{cases} \tag{3.23}$$

where T_D and V_D are the D-brane tension and the space-time volume, respectively. In calculating $f_a(\Phi_0(a))$ in (3.23), the level truncation approximation was utilized.

We take the same approach to evaluate the vacuum energies for our new solutions: we would like to see how the function $f_a(\Phi_0(a))$ behaves in the present case. If it behaves differently from eq. (3.23), it implies the presence of a new vacuum.

In order to apply the level truncation analysis, let us calculate the kinetic operator in the Siegel gauge:

$$L^{(4)}(a) = \mathcal{T}(we^{h^{(4)}_a}) - q(w\,\partial e^{h^{(4)}_a}) - k\left(w\,(\partial h^{(4)}_a)^2\,e^{h^{(4)}_a}\right), \tag{3.24}$$

where use was made of notations,

$$\mathcal{T}(f) = \oint \frac{dw}{2\pi i} f(w) T(w), \quad k(f) = \oint \frac{dw}{2\pi i} f(w). \tag{3.25}$$

The first two terms of (3.24) is easily calculated as

$$
\mathcal{T}(w e^{h_a^{(4)}}) - q(w \, \partial e^{h_a^{(4)}}) \quad = \quad \left(1 + \frac{5a}{4}\right) L_0 + \frac{a}{2}(L_2' + L_{-2}') - \frac{a}{8}(L_4' + L_{-4}'). \tag{3.26}
$$

The primed operators L_n' denote the twisted ghost Virasoro operators: $L_n' \equiv L_n + n\, q_n + \delta_{n,0}$. The Laurent expansion (3.7) leads to

$$
\partial h_a^{(4)}(w) = -4i w^{-1} \sum_{n=1}^{\infty} (x^n + y^n) \sin(2n\sigma), \tag{3.27}
$$

where $w = e^{i\sigma}$. So, it follows that

$$
\left(\partial h_a^{(4)}(w)\right)^2 e^{h_a^{(4)}(w)} \quad = \quad \partial h_a^{(4)}(w) \, \partial e^{h_a^{(4)}(w)}
$$

$$
= \quad 32 a w^{-2} \sum_{n=1}^{\infty} (x^n + y^n) \sin(2n\sigma) \sin^3 \sigma \cos \sigma. \tag{3.28}
$$

Substituting this into (3.24), we calculate the last term as

$$
-k\left(w\,(\partial h_a^{(4)})^2 \, e^{h_a^{(4)}}\right) = 2a\left\{2(x+y) - (x^2 + y^2)\right\}. \tag{3.29}
$$

Now, we parameterize x and y as

$$
x = \frac{t(1-t)}{1+t}, \quad y = \frac{t(1+t)}{1-t}. \tag{3.30}
$$

Then, using $\xi = t^2$, we obtain the final expression for the kinetic operator:

$$
L^{(4)}(a) \quad = \quad \left(1 + \frac{5a(\xi)}{4}\right) L_0 + \frac{a(\xi)}{2}(L_2' + L_{-2}') - \frac{a(\xi)}{8}(L_4' + L_{-4}') + \alpha(\xi), \tag{3.31}
$$

where $a(\xi)$ and $\alpha(\xi)$ are given by

$$
a(\xi) \quad = \quad \frac{8\xi(1-\xi)^2}{(1 - 6\xi + \xi^2)^2},
$$

$$
\alpha(\xi) \quad = \quad \frac{32\xi^2(5 + 2\xi + \xi^2)}{(1 - 6\xi + \xi^2)^2}. \tag{3.32}
$$

As ξ changes from -1 to $3 - 2\sqrt{2}$, $a(\xi)$ varies from $-1/2$ to $+\infty$.

Under the Siegel gauge condition, the energy density is given by [13]

$$
f_a(\Phi) = 2\pi^2 \left(\frac{1}{2}\langle \Phi, \, c_0 L^{(4)}(a)\Phi\rangle + \frac{1}{3}\langle \Phi, \, \Phi * \Phi\rangle\right). \tag{3.33}
$$

At level $(0,0)$ truncation, the component field is $t\, c_1 |0\rangle$, and the energy density is

$$
f_a(t) \quad = \quad 2\pi^2 \left(-\frac{1}{2}\lambda(\xi)\, t^2 + \frac{1}{3}\left(\frac{3\sqrt{3}}{4}\right)^3 t^3\right),
$$

$$
\lambda(\xi) \quad = \quad 1 + \frac{5a(\xi)}{4} - \alpha(\xi). \tag{3.34}
$$

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$\lambda(\xi)$ has two real roots $\xi^{+} = 0.0759112\cdots$ and $\xi^{-} = -0.0933769\cdots$, and these correspond to $a(\xi^{+}) = 1.89932\cdots$ and $a(\xi^{-}) = -0.362772\cdots$, respectively. We find that $\lambda(\xi) > 0$ if $\xi^{-} < \xi < \xi^{+}$ and $\lambda(\xi) < 0$ if $\xi < \xi^{-}$ or $\xi > \xi^{+}$. Then, the energy density has a local minimum as follows

$$
f_a(t_0) = \begin{cases} -\dfrac{\pi^2}{3}\left(\dfrac{4}{3\sqrt{3}}\right)^6 \lambda(\xi) & (\xi^{-} \leq \xi \leq \xi^{+}) \\ 0 & (-1 \leq \xi < \xi^{-} \ \text{ or } \ \xi^{+} < \xi < 3 - 2\sqrt{2}). \end{cases}
\tag{3.35}
$$

We proceeded up to the level $(6, 18)$ and evaluated the local minimum of the energy density. The resulting vacuum energy is depicted in Fig. 1. We observe that the vacuum energy approaches to the function (3.23) as the truncation level is increased.

The result in this section, together with the cohomology analysis, is consistent with the statement that the non-trivial solution with the forth order zeros describes the same tachyon vacuum as other non-trivial solutions obtained earlier.
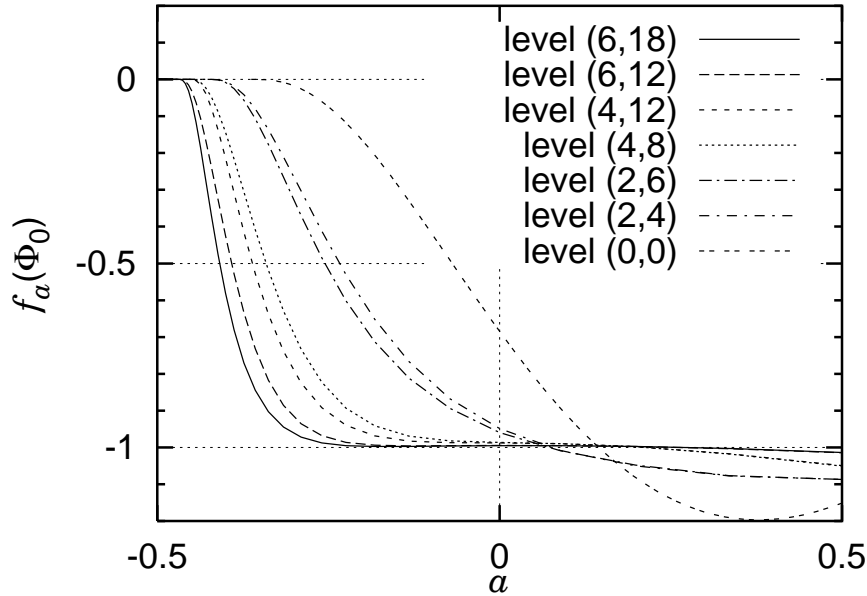

Figure 1: Vacuum energy.

4   Summary and outlook

In the construction of classical solutions to CSFT, each solution is specified by an underlying holomorphic function $F(w) = e^{h(w)}$. Because of the inversion relation $F(w) = F(-1/w)$, the same numbers of zeros are distributed inside and outside the unit circle $|w| = 1$. The solutions

with functions whose zeros are located off the unit circle are pure gauge, while the non-trivial solutions, that cannot be gauged away, are obtained with functions with zeros all on the unit circle $|w| = 1$. The functions $F^m_{-1/2}(w) = \exp\left(h^m_{-1/2}(w)\right)$ where $h^m_a(w)$ are given in (2.8) have second-order zeros. In [14], a conjecture has been made that the order of zeros is related to the number of D-branes and therefore different orders of zeros will correspond to different vacua of the theory.

In this letter, we have constructed a non-trivial solution specified by a function with the fourth order zeros. The cohomological argument on the new BRS charge shows the absence of open string excitations at the perturbative level. Our numerical analysis on the vacuum energy indicates that the non-trivial solution with the fourth order zeros describes the tachyon condensed vacuum. From these results we conclude that our new non-trivial solutions correspond to the same tachyon vacuum described by the solutions reported earlier [10, 11] and they are counter examples to the above conjecture.

We have reported yet another analytic solutions to the CSFT. Our results suggest that all these classical solutions as well as those obtained earlier are sound candidates describing the same tachyon vacuum. This is consistent only if these solutions are gauge equivalent. We will report how they could be related each other in the forthcoming paper [18].

Acknowledgements

References

[1] A. Sen, "Tachyon Dynamics in Open String Theory", hep-th/0410103, and references therein.

[2] K. Ohmori, "A Review on Tachyon Condensation in Open String Field Theories", hep-th/0102085, and references therein.

[3] A. Sen, "Descent Relations Among Bosonic D-branes", Int. J. Mod. Phys. **A14** (1999) 4061 [hep-th/9902105].

[4] A. Sen, "Universality of the Tachyon Potential", J. High Energy Phys. **12** (1999) 027 [hep-th/9911116].

[5] E. Witten, "Non-Commutative Geometry and String Field Theory", Nucl. Phys. **B 268** (1986) 253.

[6] V. A. Kostelecky and S. Samuel, "On a Nonperturbative Vacuum for the Open Bosonic String", Nucl. Phys. **B 336** (1990) 236.

[7] A. Sen and B. Zwiebach, "Tachyon Condensation in String Field Theory", J. High Energy Phys. **03** (2000) 002 [hep-th/9912249].

[8] N. Moeller and W. Taylor, "Level Truncation and the Tachyon in Open Bosonic String Field Theory", Nucl. Phys. **B538** (2000) 105 [hep-th/0002237].

[9] D. Gaiotto and L. Rastelli, "Experimental String Field Theory", J. High Energy Phys. **0308** (2003) 048 [hep-th/0211012].

[10] T. Takahashi and S. Tanimoto, "Marginal and Scalar Solutions in Open Cubic String Field Theory", J. High Energy Phys. **03** (2002) 033 [hep-th/020133].

[11] I. Kishimoto and T. Takahashi, "Open String Field Theory around Universal Solutions", Prog. Theor. Phys. **108** (2002) 591 [hep-th/0205275].

[12] T. Takahashi and S. Zeze, "Gauge Fixing and Scattering Amplitudes in String Field Theory Expanded around Universal Solutions", Prog. Theor. Phys. **110** (2003) 159 [hep-th/0304261].

[13] T. Takahashi, "Tachyon Condensation and Universal Solutions in String Field Theory", Nucl. Phys. **B670** (2003) 161 [hep-th/0302182].

[14] N. Drukker, "On Different for the Vacuum of Bosonic String Field Theory", J. High Energy Phys. **08** (2003) 017 [hep-th/0301079].

[15] S. Zeze, "World Sheet Geometry of Classical Solutions in String Field Theory", Prog. Theor. Phys. **112** (2004) 863 [hep-th/0405097].

[16] M. Kato and K. Ogawa, "Covariant Quantization of String Based on BRS Invariance", Nucl. Phys. **B212** (1983) 443.

[17] I. B. Frenkel, H. Garland, G. J. Zuckerman, "Semiinfinite Cohomology and String Theory", Proc. Nat. Acad. Sci. 83 (1986) 8442;
M. Henneaux, "Remarks on the Cohomology of the BRS Operator in String Theory", Phys. Lett. **B177** (1986) 35.

[18] Y. Igarashi, K. Itoh, F. Katsumata, T. Takahashi and S. Zeze, in preparation.